# Day 15

Kinematics of Wheeled Robots

# Wheeled Mobile Robots

- robot can have one or more wheels that can provide
  - steering (directional control)
  - power (exert a force against the ground)
- an ideal wheel is
  - perfectly round (perimeter  $2\pi r$ )
  - moves in the direction perpendicular to its axis

# Wheel





#### **Deviations from Ideal**

This illustration gives a good sense of the steering and throttling you'll have to input to keep your car drifting. When snapping the car from its full drift angle in one direction to full drift in the opposite direction, be prepared for the rear end to come around with more force 1 then when initiating a drift from straight-ahead running. Give yourself plenty of space as you master your technique so you don't slap a curb or something equally immobile!

Steer hard into the turn to initiate a slide, then countersteer before the car loops out.

2 Continue countersteering to maintain the slide. It's a balancing act!

ting up for the next turn. The goal is to drift all the way through without straightening out for more than a moment as the rear end swings around.

drift!



You'll have burned off some speed by now, so be prepared to pin the throttle to keep the rear wheels slipping. Traction kills

3

3 Now you're set-

2

5 Here's where it gets tricky. Keeping the rear end sliding through turns is relatively easy, but drifting down a straight is tough. As you lose momentum, the car will straighten out, but you'll be surprised how long you can hang it out. Good luck!

http://worlddrifting.com/wp-content/uploads/2010/02/drifting-techniques.jpg 4

## Instantaneous Center of Curvature

- for smooth rolling motion, all wheels in ground contact must
  - follow a circular path about a common axis of revolution
    - each wheel must be pointing in its correct direction
  - revolve with an angular velocity consistent with the motion of the robot
    - ▶ each wheel must revolve at its correct speed

#### Instantaneous Center of Curvature



(a) 3 wheels with roll axes intersecting at a common point (the instantaneous center of curvature, ICC).
(b) No ICC exists. A robot having wheels shown in
(a) can exhibit smooth rolling motion, whereas a robot with wheel arrangement
(b) cannot.

Dudek and Jenkin, "Computational Principles of Mobile Robotics"

## **Castor Wheels**

provide support but not steering nor propulsion



# Differential Drive

two independently driven wheels mounted on a common axis



# Differential Drive

velocity constraint defines the wheel ground velocities



# Differential Drive

given the wheel ground velocities it is easy to solve for the radius, R, and angular velocity ω

$$R = \frac{\ell}{2} \frac{\left(v_r + v_\ell\right)}{\left(v_r - v_\ell\right)}$$
$$\omega = \frac{\left(v_r - v_\ell\right)}{\ell}$$

## Tracked Vehicles

- similar to differential drive but relies on ground slip or skid to change direction
  - kinematics poorly determined by motion of treads



http://en.wikipedia.org/wiki/File:Tucker-Kitten-Variants.jpg

#### Steered Wheels: Bicycle



# Steered Wheels: Bicycle

- important to remember the assumptions in the kinematic model
  - smooth rolling motion in the plane
- does not capture all possible motions
  - http://www.youtube.com/watch?v=Cj6hol-G6tw&NR=l#t=0m25s

### Mecanum Wheel

a normal wheel with rollers mounted on the circumference



http://blog.makezine.com/archive/2010/04/3d-printable-mecanum-wheel.html

http://www.youtube.com/watch?v=CeeIUZN0p98&feature=player\_embedded

# Mecanum Wheel

Direction of	
<u>Movement</u>	Wheel Actuation
Forward	All wheels forward same speed
Reverse	All wheels backward same speed
Right Shift	Wheels 1, 4 forward; 2, 3 backward
Left Shift	Wheels 2, 3 forward; 1, 4 backward
CW Turn	Wheels 1, 3 forward; 2, 4 backward
CCW Turn	Wheels 2, 4 forward; 1, 3 backward

**To the right:** This is a top view looking down on the drive platform. Wheels in Positions 1, 4 should make X- pattern with Wheels 2, 3. If not set up like shown, wheels will not operate correctly.



AndyMark Mecanum wheel specification sheet http://dlpytrrjwm20z9.cloudfront.net/MecanumWheelSpecSheet.pdf

- serial manipulators
  - given the joint variables, find the pose of the end-effector
- mobile robot
  - given the control variables as a *function of time*, find the pose of the robot
    - for the differential drive the control variables are often taken to be the ground velocities of the left and right wheels
      - it is important to note that the wheel velocities are needed as functions of time; a differential drive that moves forward and then turns right ends up in a very different position than one that turns right then moves forward!

robot with pose [x y θ]<sup>T</sup> moving with velocity V in a direction
 θ measured relative the x axis of {W}:



• for a robot starting with pose  $[x_0 \ y_0 \ \theta_0]^T$  moving with velocity V(t) in a direction  $\theta(t)$ :



$$x(t) = x_0 + \int_0^t V(t) \cos(\theta(t)) dt$$
$$y(t) = y_0 + \int_0^t V(t) \sin(\theta(t)) dt$$
$$\theta(t) = \theta_0 + \int_0^t \omega(t) dt$$

for differential drive:

$$x(t) = \frac{1}{2} \int_0^t (v_r(t) + v_\ell(t)) \cos(\theta(t)) dt$$
$$y(t) = \frac{1}{2} \int_0^t (v_r(t) + v_\ell(t)) \sin(\theta(t)) dt$$
$$\theta(t) = \frac{1}{\ell} \int_0^t (v_r(t) - v_\ell(t)) dt$$

# Sensitivity to Wheel Velocity

